**Brief Lecture Note on Representation and Discriminative Prediction of Topological Information Structures (re: EMME, Seldon, predictive analytics, tensor algebras)**

(text notes here, mainly - graphics, charts, equations, etc. are primarily in other docs or meant for "live" presentation time)

MJD (updated) 01.sept.22

**[§ 1]**

We have been discussing the use of topological models in n-space for representation of dynamic processes including causal relationships, hypothesis formation and the kind of abductive reasoning required to evaluate alternatives such as in the EMME space. So we are looking at how we can employ geometry to do things like represent a configuration of agents or forces acting on each other in a manner that causes changes in some of their attributes which are going to be nodes in a network, perhaps even a DAG (directed acyclic graph), that geometrically is something like a knot or a twist, some type of deformed sphere or torus. Then we want to examine the topology of these things and seen how as manifolds they can change over time and as differences are applied, think of them as forces, to act upon the surfaces.

That gives us a path toward having an algebraic method (this gets back to process algebras and some of that work we discussed from Hoare and May and others in a purely computational context) which we can use in the analysis of these topologies for how they represent configurations of data and how they can interact and change - and also how they can be classified, recognized, identified. and then we can get into the problem of how to make predictions about what will likely happen - the what-ifs - when some of these topological structures are affected over time by whatever will modify some of their parameters. Ultimately that gets us into the application-space of things like EMME, where there are changes in climate, or atmospheric conditions, or the type of vegetation or the animals that feed on such, and then we can see how likely it may be - given some of those changes - for some other events to occur. Such events may be in the micro-biosphere, with viruses and bacteria, including a lot of types that are either omni-present but dormant, not affecting their hosts for instance in a significant way (e.g., Yersina pestis), or which are present but which will change through some mutation as a result of the altered biosphere into a form that becomes affective in a pathogenic manner. We want in these situations to evaluate a sizable and potentially huge number of competing hypotheses in order to find those that fit the data best and are most worthwhile to investigate further, including empirically.

Of course, the applications can be totally different, all sorts possible, even in the socioeconomics of marketing and public interests and preferences, but ultimately we are talking about some type of environment, a space with a large number of different "species" that inhabit that space and interact, affecting the populations, as it were, of these different "species", and all subject in some ways to common forces which in the natural world and what we usually mean about "environment" have to do with the elementals that matter - air, water, earth, the temperature, humidity, chemical compounds therein, etc. With the geometric-topological approach we are undertaking, our goal is to have a richer mathematical toolset for identifying patterns and relationships among objects that we create in an abstract space but which are connected with empirical observations - or simulated observations, where the latter can be widely variant from from we actually measure empirically but which are realistically possible.

**[§ 2]**
The point about a geometric-topological form of representation is that we can develop a method for comparing many different perspectives using abstract forms (e.g., curvatures, volumes, holes, knots, twists) in these objects, and we can study their interactions using a model based somewhat on physics - if not actual physics as it exists in the observed world, then a possible type of physics, and one that can have consistent rules and formulae that we apply to everything in our model-world.

We need a versatile, flexible way to represent a field of potentials, for instance, which act upon the objects in our n-space world that represent processes in the natural world. The metaphor here being exactly that from classical electromagnetics - forces that act upon some entity, in this case a relational object, and change change its behaviors, its properties. In a simple abstract model we can think of a space filled with bouncy inflated balls, soft and easily deformable, and let's say that they change in color and brightness when they get deformed by forces from all the balls around them, and not necessarily limited to those directly touching each other. We want some tools, better than what have been mostly tried and used thus far, like in conventional "AI" and neural networks that are mostly reducible to matrix representations, everything ending up in 2D arrays and then all the usual weightings and statistics, etc. Moreover, we want to pay close(r) attention to the dynamics of change - the equivalents of velocity, momentum, acceleration, even functions that can be compared and modeled upon charge and gravity.

In our world of colored balls, we would like to be able to predict when and where in our ball-space there's going to be a ball that maybe gets squeezed and compressed enough so that it turns bright red or yellow, or pops, or gets squeezed into some singularity, because such changes would indicate something significant, meaningful, to the system as a whole, perhaps some new emergent condition, some type of phase transition, something that we could say is going to change the whole system, the network of all these balls. And we'd like to be able to notice whatever we can, early on, before those bigger or perhaps irreversible changes occur, in order to perhaps do something that will alter our network, such as to prevent the outcome or in some cases hasten it or otherwise to have some control over what is happening, even if that "control" is limited to having new information, knowledge, that has some value to us for using as we stand, as it were, 'outside" the system as observers.

But mostly, in the real-world applications where we want such knowledge, it's because we want to actually do something - to have some active control, not just passive knowledge. If there is a series of events that can lead to a pandemic emerging from a new type of virus, we'd (presumably!) want to use our knowledge of that in a way that can help us to prepare, to prevent it or in some way adapt to it as best we can.

**[§ 3]**
As we've been discussing, that's what the EMME Project is all about. Predicting some type of emerging change that relates to pathogens that can affect humans or special species that matter to us, such as in agriculture including livestock for our food supplies.

We can use the example of transitions in the biosphere that lead to migrations or some other changes that affect the metabolism of one or more species, in turn leading to emergence in that region (locality where such changes occur) of pathogens that are risks to other species including humans. We have a wealth of examples from recorded history on which to draw from for examples and to build our set of hypotheses that we will want to see how they fit with current observables from our present world.

I will use the example of the Plague of the 6th century CE, which decimated Constantinople and then spread throughout the Mediterranean and much of Western Europe including to the British Isles. This is covered in a number of publications and in that video I mentioned previously [1].

Historically, there is reasonable evidence that the plague of that era was a form of the bubonic plague, thus the Yersina pestis bacteria, and it is interesting that it could have emerged from sources in East Africa where it had been present but relatively inactive because of the metabolism of the bacteria and how it was interacting with the host fleas - until there was a significant change in the climate, namely a cooling down which would have followed from the Krakatoa volcanic eruption and its effects on the global climate.

For the moment, let's just make some assumptions for our model exercise here. This is not about determining everything that did happen 1,500 years ago but in examining a kind of "use-case" situation as an illustration. What we can gather from the historical record (cf. some comments in note [1] below) is that we had a situation where there was relative stability within a network-space of processes that we can visualize as the soft bouncy balls and ecologically as a world where there are humans in communities, interacting, trading, and an agricultural system that is producing more or less sufficient food for everyone, and also the usual other species including rats, and the fleas for which the rats are hosts, and the bacteria that feed on the substances inside the guts of the fleas. And so on. Reasonable stability.

Then along comes drastic change in the overall physical environment, and faster than what we have been seeing, in modern times, with climate change that is in the other direction, namely warming and with it, cycles of extremes in weather patterns and especially affecting precipitation and surface water. These drastic changes back in the 6th century CE are, in terms of our geometric-topological model building, like this:

- in our abstract ball-world model, one ball gets squeezed and contorted by the forces coming from others in the "connected network" so that it "pops" and this leads to consequences that affect many other balls in the system, leading to a cascade of other balls being strongly distorted, contorted, deformed.

- in the ecological world, the overall temperature drop leads to changes in the Yersina pestis bacteria within their host fleas, such that there are physical blockages in their digestive tracts, leading to poor digestion of whatever the fleas have as their intake through blood from their hosts, leading to constant and insatiable hunger, leading to jumping onto all sorts of other nearby victims including humans, and thus infection into new species (e.g., humans) with the bacteria, leading to bubonic plague pathogenesis.

But just as the ball-space has a lot of interconnected balls, so the real-world eco-space has a lot of other connections among the localized informational entities which we want to model in a manner that allows us to treat those, mathematically and computationally, as geometrical objects and with interesting topologies that can tell us, by their changes, something important (we hope and intend!) about how they are changing in shape and what to expect as future changes in shape.

Some of these entities are about information concerning the rest of the ecosystem, such as:

- water available and used for agriculture and for human and domesticated animal consumption (declining because of droughts in this historical period - and btw this has been historically shown for many parts of the world at this time)
- food supply in general (also in decline due to the droughts and the reduced sunlight because of dust clouds from the volcanic eruption)
- general decline in health among the human population thus affected
- decline in hygiene in the homes and communities, due to all the above here
- reduced quality of air intake and overall nutrients
- increased conflict including violence, war, breakdown of social structures

So all of these constitute what can be modeled as dynamical geometrical structures whose shapes are affected by the forces acting upon them, namely, the forces from other structures in the network, what we can call the topological informational relations (TIR) space or simply the topological information space (TIS).

**[§ 4]**

We propose that there can be a mathematical approach to modeling all these behaviors and it is something well-known at least in some areas of science, especially physics, but less used to date in the areas of knowledge engineering and "machine learning".

Tensors for describing the nodes in a TIR structure and their relations with active nodes in a tensegrity network - this is our goal. The tensors need to be such that they can represent the gradients, the differentials of changes in the topology of each node and we need to be able to map those changes in some node $n_{[i]}$ with a set of other nodes in the network which are $n_{[j \neq i]}$.

Each TIR node will describe some entity in an n-dimensional manner. What are those dimensions? It will depend upon what we are modeling, of course. But we must allow for some nodes having different dimensions from others. That means we will be dealing with comparing objects (nodes) that have different dimensionality - a challenge indeed. (How does one "compare" triangles and cubes?)

**[§ 4.1]**

First, some review for the purpose of both introductory overage to some terms and for attaining some consistency, because of the massive mixing and confusion about terms regarding tensors in much of the literature, especially in recent years in the "machine learning" communities. Here, we'll make some simple definitions. These will agree with some but not all people working in the field!

Rank, axis, dimension, order, length and shape are terms that come up in any operations involving tensors and these get used in "every which way" with in the software community, as far as I can see.

For our purposes here and going forward (subject to change, perhaps!):

Let's begin with dimension, which we are going to use synonymously with order. Dimension tells us what is required, or possible, for defining the position of some object within a coordinate space. Thus

(x) for a position on a line, (x,y) for position on a [plane (2D), (x, y, z) for position in 3-space, and so on. A tensor will be of the dimensionality (order) corresponding to what one needs (or can have) for specifying something within the space in which it exists or operates.

A vector [1,2,3,4] has 4 dimensions or its order = 4 because there are 4 elements, each specifying a coordinate position within one of the dimensions. This should not be confused with thinking of the dimensions, or the position-values therein, as being only like locations in the usual coordinate systems we use, because as we move from what we could describe as a physics-mainly perspective to a general informational perspective, these dimensions can be widely different in how we represent "locations" within them!

Matrices as collections of vectors, such that any one row in the matrix describes one point in a vector space. The vectors are all the same length (each trow with the same number of elements).

Example:

```
m = [ [2,3,4,5]
      [8,7,6,5] ]
```

The number of rows (2) gives the number of points you are describing in that matrix m, and the number of columns (4) gives the number of dimensions you are working with.

Tensors are from one perspective any collection of ordered elements and thus a scalar (e.g., 12), a vector [2,3,4,5] and a matrix like m above are all tensors, but in general usage the term tensor is given to 3D and higher dimensionality (order) structures.

Thus, for example:

```
t = [ [2,3,4], [5,6,7], [6,7,8]        dimension "a"
      [0,1,2], [4,5,6], [7,8,9]        dimension "b"
      [4,3,2], [7,6,5], [9,8,7]        dimension "c"
    ]
```

Dimensionality = 3 = order = 3.

It seems that most people are using rank in the same general sense as dimension and order, but I will suggest that rank specifically is about how many indices are required to specify individual elements of a tensor from its numerical set, be that a vector, matrix, cube or higher dimensionality. A vector requires one index (position in the row) and is a rank 1 tensor. A matrix requires two indices (row, column), and is thus a rank 2 tensor. A cuboid tensor requires 3 indices (row, column, depth (third direction) and is thus a rank 3 tensor. And so on.

To me, these are different terms for aspects of the tensor that are going to be the same. Three dimensions means you need three indices, and vice versa, and there will be three axes involved, each axis corresponding to one dimension

Rank, axis, dimension, order we will use as referring to the number of dimensions, thus the number of axes in any numerical, computational representation. [2]

Length of a dimension is the number of elements along that dimension, on that axis, that are in the tensor. Length is how many values we can have along an axis. But depending upon what one is working with, and what the dimension, measured on some axis, means in the context of what the tensor is employed to represent, the length could be various. The length of each axis tells us how many index values are available along each axis, not some value of distance along that axis! Remember that the dimension is not simply or always a line of distance in either real or complex number space and there could be a particular finite number of values possible, from which to choose, for assigning a value to use in a particular tensor.

Consider that one dimension is "humidity" and we are not measuring a specific value from 0 to 100% or some specific mm of rainfall but ranges, corresponding to some qualitative values such as "extremely and drastically arid", "arid", "semi-arid", "mild", "wet", "constantly wet". So in this case the length along the xis for humidity would be 6.

Shape is a way of describing the set of lengths for the different dimensions. Shape is a tuple of [a, b, ...] where these elements are the lengths (as numbers of elements) of each axis in the tensor. In a way, one can think of the shape of a tensor as describing that it is a composite of lengths (numbers of elements) of the different dimensions.

Example:

```
t1 = [1,2,3,4,5]          dimension a (columns)          Shape of t1 = [5]


t2 = [ [2,3]              dimension a (columns)          Shape of t2 = [3,2]
       [5,6]
       [6,7]              dimension b (rows)
     ]

t2a = [ [2,3,4]           dimension a (cols)             Shape of t2a = [3,3]
        [5,6,7]
        [6,7,8]           dimension b (rows)
      ]

t3 = [ [[2,3], [5,6], [7,8]]      dimension a (each row member)       Shape of t3 = [4,3,2]
       [[0,1], [3,5], [6,9]]      dimension b (cols)
       [[4,2], [6,5], [8,7]]
       [[8,6], [5,4], [2,1]]      dimension c (rows)
     ]

t3a = [ [[2,3,4], [5,6,7], [6,7,8]]       dimension a (each row member)       Shape of t3a = [4,3,3]
        [[0,1,2], [4,5,6], [7,8,9]]       dimension b (cols)
        [[4,3,2], [7,6,5], [9,8,7]]
        [[0,1,2], [4,0,5], [6,7,0]]       dimension d (rows)
      ]
```

The shape of a tensor is a tuple (ordered list of numbers) whose length (number of elements) is the order of the tensor and each of its elements are the lengths of the dimensions of each axis. Shape of the tensor is important in how things can be transformed for different computations using the tensor and certainly this is expected to enter into the topological computations that we expect will ensue, some of which are described further below.

For instance, even with considering tensors in the more "traditional" way as n-dimensional arrays, there will be times when we need to perform "reshaping". This can be shown by the following simple example:

```
Consider t1 = [ [1,2,3]        Rank = 2 and shape = [3,3]
                [4,5,6]
                [7,8,9]
              ]
```

But suppose we need to transform this t1 → t2 with shape = [1,9]?
t2 = [ [1,2,3,4,5,6,7,8,9] ]

The shape may change but not the constituent elements.

For where we are going with TIR (TIS), this is likely going to be significant, as we will have a few interesting problems to solve operations that involve:

- comparisons including segmentations between node-objects that are represented by tensors of different dimensionalities and thus different shapes
- estimates regarding curvatures, twists, knot-like formations, and other interesting topological properties that can be employed to indicate degrees and rates of deformation which are linked to some underlying phenomena represented by the objects (e.g., rates of introduction of new species or new predator-prey or parasite-vector relationships into a particular geographic region or environmental type).

**[§ 4.2]**
Using the tensor model for representing not only the TIR node at some given instant in its "location" within its n-space, but also, and very important we will claim, for giving us a tool to represent how the TIR can change shape, under what influences (forces from other nodes in the network) and into what it can change shape - this states more comprehensively our goal in all this.

Remember that the shape of the node (not to be confused with tensor "shape" as discussed previously in [§ 4.1]!) is some type of manifold, a topological surface that in our worldview continuous and unbroken as a surface (i.e., no singularities, no breaks; examples: something that is transformable into a basic sphere or torus, no matter what its geometry at a given point in observation; thus, a tetrahedron, cube or other polyhedron can be transformed, either smoothly or non-smoothly, into a sphere, and vice versa, and a donut-type torus can be transformed into a coffee cup with a handle, and so forth).

The parameters from our different observables (e.g., environmental attributes, whatever we have chosen to measure and put into our n-space) are what result in a set of objects with geometrical attributes, and each such object has at any given point in time (of observation, measurement) a defined topology. What we care about is how to notice when there are significant changes to some of these objects, to their shapes, that give us an idea that there is something happening in the world that these objects represent (e.g., the natural environment, the biosphere) that in some way connects - supportive or diminishing, positive or negative - with an associated hypothesis which we have formed and linked with such object topology behaviors.

Thus, for example, consider a hypothesis that begins with the proposition,

> "increase in temperature over a geographic region contiguous with a region where Trypanosoma cruzi is prevalent can lead to spread of its common vectors, the triatomine insects (genera *Triatoma*, *Panstrongylus*, and *Rhodnius*) and thus the parasite itself, leading to potential increases of infection among humans". [3]

We can consider in the TIR model that there is a tensor representing this which behaves in a way that changes according to some observations or some ranked expectations (probabilities) of the appropriate zoological, botanical, climatic, and other environmental observables. Then when for whatever reason we see changes in that tensor's topology, we consider that it is time to look into matters more closely and to probably investigate with more fieldwork, using whatever are the appropriate and available protocols (e.g., in this case, physical sampling, and a lot of social media attention and public health education, in order to enlist the population to not only be on the lookout but to take the appropriate precautions - most of which in this particular case pertain to hygiene in homes and other habitable areas).

However, we need to emphasize that whatever these tensors do, a very important role is in representation of dynamical change, in effect gradients within multi-dimensional spaces, and this brings in a lot more to do with curves, differentials, limits and boundaries for what can change and at what rates and in what directions. The TIR use of tensors is definitely about representing dynamical behaviors in regions of our information space, and we do not want to think of the tensors as being "static" representations. This is important because in the way that tensors have been often used within neural network recognition and learning systems, such as with images, text, and other artifacts from the observable world, those artifacts are often in themselves fairly static; i.e., unchanging over time, space, or in relation to other objects.

**[§ 4.3]**

Thus, one challenge with the TIR modeling and the computation to support it, for applications such as EMME, is that we need to do pattern discrimination, recognition, classification and prediction on the basis of how these entities - represented by some tensor configurations - change and are likely to change. Such tasks are really the whole purpose of the computational framework and any "synthetic intelligence" algorithms. We want to be able to rapidly evaluate many possible transformations that can occur within the tensor-represented objects, the nodes, of a network with many interconnects between them that can have causal significance, and which we cannot hope to enumerate by our own human-only associations and hypothesis-building, and which we cannot hope to evaluate in real-time given the number of changes that can occur to the nodes in our network (e.g., in the context-example of EMME, changes in monitored environmental conditions that originate from data coming in from satellites and earth-based sources, both automated and manual).

We want to be able to compare tensors at different points in time and especially in how they transform in some functions (this gets back to our "sea" of hypotheses for evaluation, the basic abductive reasoning problem) which involve interactions between the different topological objects that are represented by these tensors.

I have been talking before about such dynamics as chaotic and stochastic (aka "strange") attractors, which are dynamical systems and typically of a dissipative nature (e.g., tornado, hurricane) such that some driving force keeps the system 'alive" and with a tendency toward some typical behavior (the "attractor"). But also in our discussions and the picture of the world that we are building is the concept of a soliton, as a process with non-dissipative and form-retaining wave-like dynamics. These seem to be at opposites but perhaps not as much as how they have been regarded and separated in past studies.

Can there be, within a network of nodes that represent physical (and biological) processes, some type of attractor behavior which has both stability under certain conditions but also certain instabilities that can throw the attractor dynamics into a radically different form (and thus topology in our way of representing things), and is there some type of stabilizing dynamical behavior within the network, some energy exchange as it were between the nodes, which provides some of that "driving force" for the attractor but in a manner that can keep things stable in terms of its attractor form? And if that "harmonic soliton-like wave behavior" within the network is disrupted, shifted, can it create destabilizations that then cause the attractors to undergo disruptive changes, some of which - getting back to what those processes are (may be) in the natural world - lead to the kind of disruptive states that we want to predict in advance of their happening (e.g., pathogen-vector changes that lead into epidemic outbreaks, etc.).

This is where these things come into the picture within the TIR model space. If we can identify such behaviors which demonstrate at the same "time" (meaning, here, in the same dynamics of changing state-spaces which are being represented in our network) randomness and non-linearity (such as bifurcation points), but also some overall stability. Then we may be on track to having a better way of understanding the kinds of phenomena that we observe so often in the natural world, not only but especially in the space of climate and environment.

*See the sampling of some attractor geometries and inferrable topologies in note [4], as a preview of some things to come.*

I believe that this whole area of "stability within highly nonlinear spaces" is very important for may fields and for wherever we are thinking and working toward building synthetic intelligence; i.e., computational engines that can find associations, relations, including dependencies, causalities, even synchronicities that may not be explainable in causal forms, at least given our present knowledge.

But about attractors both chaotic and "strange" as well as multi-dimensional solitons, this is for a future talk. We don't need all this right now. We just need to build the proper foundations - mathematically and algorithmically - for accommodating such "wilder and more exotic" things in the future. We need to have ways to measure changes in curvature and 2nd and 3rd derivative type rates of change that are in a network of objects exerting forces upon one another. Then we can consider looking for unique patterns emerging in those behaviors.

Remember - we want to be able to make predictions about how the topologies of our objects will change. Getting back to our primitive illustration of a world of soft balls that influence each others' shapes by collisions or forces at some distance, we want to show how certain balls will be deformed, in what angles and with what curves, and how they may move around in their space. Thus we will be able to generate simulations that project such geometrical changes, leading to configurations that can appear strongly to be indicative of a system-level change which merits attention.

That attention comes in the form of further analysis of the simulated consequences, a combination of human and machine reasoning. Ultimately this turns into "pruning the tree of possible branches" and gives us indications of the most probable and most significant branches which can be either positive or negative in value for the system as a whole.

**Notes**

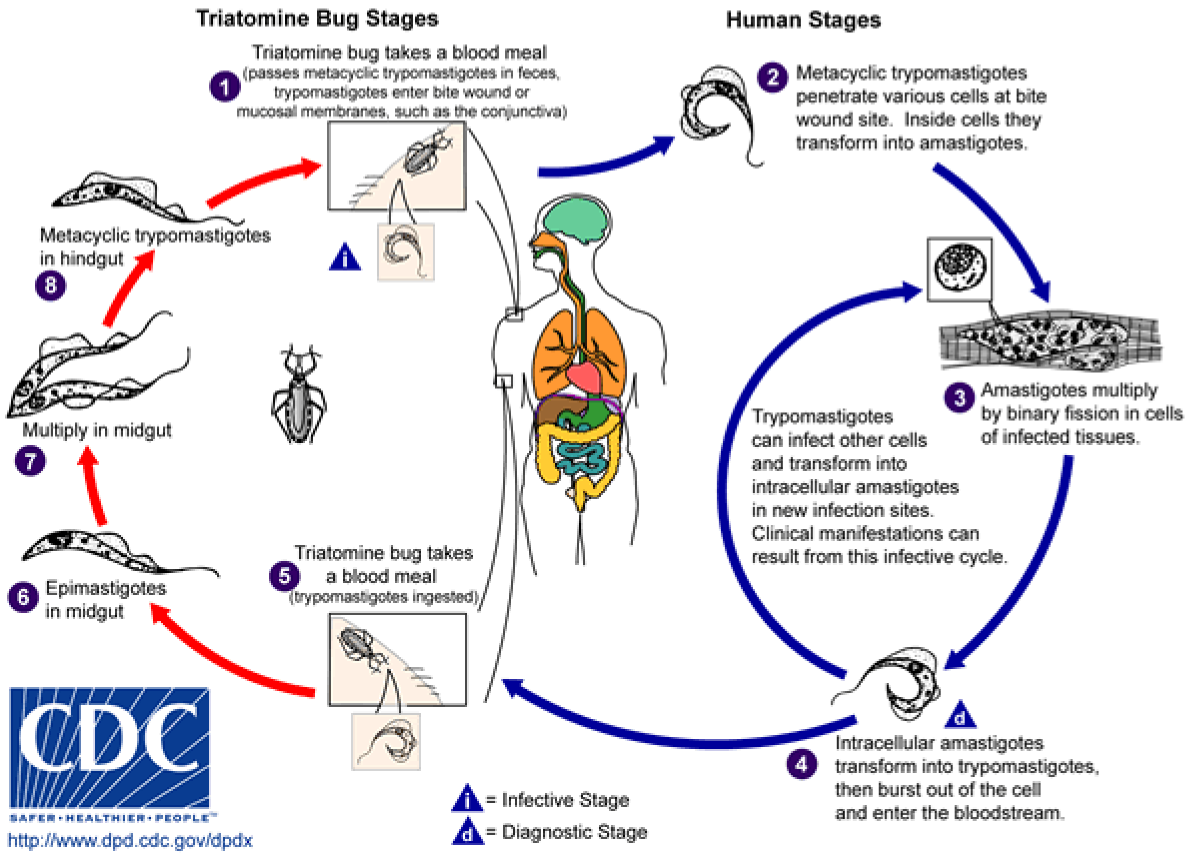
[1]
https://www.youtube.com/watch?v=Ax2x0MG-0Qo
536 AD: The Worst Year In History? | Catastrophe
Description of the Plague @ mid-6th century begins @ 56:45 into the broadcast.
Caveats to the viewer - it has all the usual styles and attributes of these different "history channel" broadcasts, so bear with it. The first part is mainly about historical events and looking for the evidence of something that could have drastically affected climate by a planetary-wide cooling down, and then looking for what could have caused that, with a good argument for it being a volcano and one that likely had a severe eruption during the general window of time including 6th century CE. One of the points here in bringing this up is that there as a severe pandemic that was recorded by many observers in many parts of Europe, and it may have been triggered by something that essentially 'tipped the scale" for a relatively dormant pathogen to suddenly become more active in terms of contagion and transmission and infection among humans.

It is also interesting to compare this historical pandemic with the more well-known bubonic plague of the 14th century and its origins which were (apparently, at least to my understand thus far) not climate-related but connected with movements - dislocations - of the catalyst species, rats and their fleas, and their introduction into human environments that were primed and just right for generating first an epidemic and then a pandemic, namely grossly unhygienic, filthy conditions of people in siege warfare followed by dispersion throughout Europe, starting in the Mediterranean, by refugees and traders onboard ships.

[2]
There seems to be enough consistency in this view of things tensorial, although in the web vernacular among the "AI" community, as found on many websites, there is anything but agreement on these points.

[3]
Chagas disease is caused by infection with the parasite *T. cruzi*, a protozoan, whose vectors are known to include the various triatomine bugs, also called "kissing bugs". The insect defecates at the bite site, and from the insect feces, trypomastigotes (motile forms of *T. cruzi*) enter the bloodstream and invade various host cells. Once inside a host cell, the parasite transforms into an amastigote, which thence undergoes several rounds of replication. These replicated amastigotes transform back into the trypomastigote form and thence burst the host cell and enter the host bloodstream, spreading

throughout the body to various tissues, where they continue the process of endocytosis and replication. The disease in humans can progress for several years through many cycles of parasite replication. The body's immune response is linked with severe damage within these affected tissues, particularly the heart and digestive tract, and the results are fatal with little prognosis for recovery unless the disease has been diagnoses and treated at an early stage. The figure below illustrates the lifecycle:



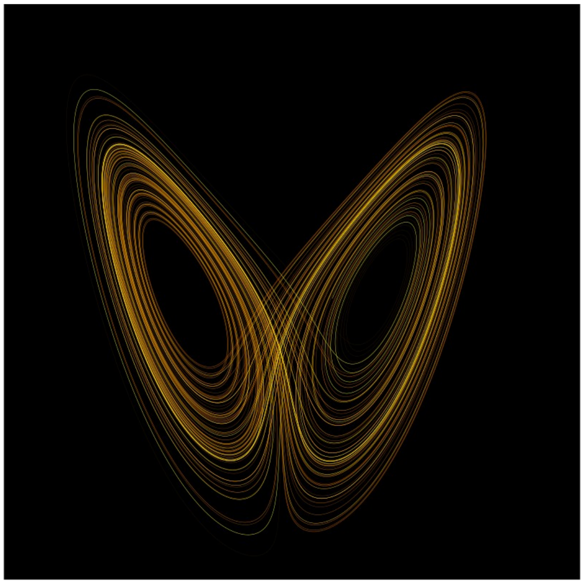

[4]

In the future I will get into everything about attractors, invariant sets, limit sets, strange attractors and their fractal nature, and how dynamical systems evolve, but not in the lecture note (which is already long enough). Here, below, are simply a few images of different types of "TIR animals" that we can expect to encounter in a TIS environment of modeling phenomena and relational rules using tensors that can be manipulated to show us (we hope!) how these objects are changing as a result of network influences.

When you examine these, try to think about the shapes of the wholes that you see - and yes, by the way, don't forget the "holes" within the wholes! Try to visualize how small changes to certain parameters that govern these dynamical systems - or even seemingly large changes - can have any of the following types of consequences:

- dramatic changes in the net iterations and the future "shape" of the process
- little or no discernible (or significant, for our system-level purposes and aims) changes!

- effects that, when translated to the "real-world" application for which such a TIS system can be engineered, result in significant-enough indicators that something is going on which merits attention in the form of new data collection and analysis - such as what we will need to do in an EMME model where there is a change in the shape of something like such an attractor that indicates a biosphere change of significance for humans and their world.



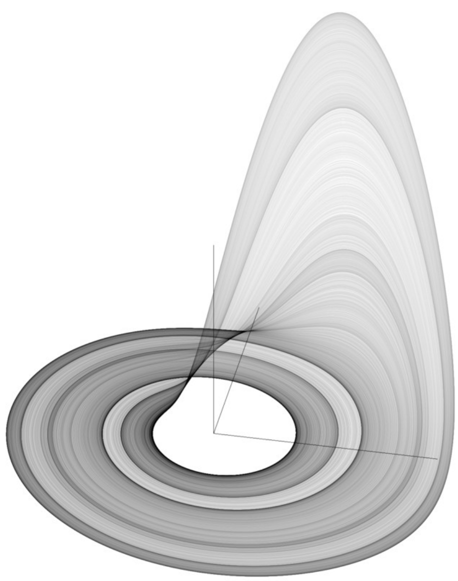
$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

Lorentz attractor, based upon 3 differential equations, with constants $\rho = 28$, $\sigma = 10$, and $\beta = 8/3$.

Another example is the Roessler attractor, here shown with a = 0.2, b = 0.2, c = 5.7:



$$\begin{cases} \frac{dx}{dt} = -y - z \\ \frac{dy}{dt} = x + ay \\ \frac{dz}{dt} = b + z(x - c) \end{cases}$$

Now [food for thought] imagine that something like one of these represents a node describing the ecosystem interactions that constitute one or more hypotheses within an EMME problem set.